First order Perturbation :-

$$a_k^{(1)} = \frac{1}{i\hbar}\int_{-\infty}^{+\infty} \langle k|H'|m\rangle e^{i\omega_{km}t}\, dt \quad \text{--- (13)}$$

Perturbation Constant in Time :-

From eqⁿ (13)

$$a_k^{(1)} = \frac{1}{i\hbar}\int_0^t \langle k|H'|n\rangle e^{i\omega_{kn}t'}dt' \qquad (14)$$

where

$\langle k|H'|n\rangle$ is valid when $0 \le t' \le t$

$$a_k^{(1)} = \frac{1}{i\hbar}\langle k|H'|n\rangle \int_0^t e^{i\omega_{kn}t'}dt' \qquad (15)$$

$$\int_0^t e^{i\omega_{kn}t'}dt' = \frac{\left(e^{i\omega_{kn}t}-1\right)}{i\omega_{kn}}$$

$$a_k^{(1)} = \frac{1}{i\hbar}\langle k|H'|n\rangle \frac{\left(e^{i\omega_{kn}t}-1\right)}{i\omega_{kn}} \qquad (16)$$

On squaring and solving — Amplitude (17)

$$\boxed{|a_k^{(1)}(t)|^2 = \frac{4|\langle k|H'|n\rangle|^2}{\hbar^2\omega_{kn}^2}\sin^2\frac{\omega_{kn}t}{2}}$$

$P_{k\to n} = P(t)$ — oscillator

Probability ground → Excited state

Resonant frequency b/w two states $T = \frac{2\pi}{\omega_k - \omega_n}$

This is the expression for transition probability when the system is in excited state.

Probability

Here $|a_k^{(1)}(t)|^2$ or $P_{nk}(t)$ is the oscillating sinusoidal func" of period $\frac{2\pi}{\omega_{kn}}$ (like oscillation).

ω_{kn} = resonant frequency b/w two states.

$$\frac{4|\langle k|H'|n\rangle|^2}{\hbar^2} = \text{amplitude}$$

$P_{nk}(t)$ will be maximum when $\frac{\sin^2(\omega_{kn}t/2)}{\omega^2_{kn}}$ is maximum.

and value of $\frac{\sin^2(\omega_{kn}t/2)}{\omega^2_{kn}} = 1$

when

$\omega_{kn} \longrightarrow 0$ then

$$\frac{\sin^2(\omega_{kn}t/2)}{\omega^2_{kn}} = 1$$

[i.e all the atoms are in ground state ∴ no condition of population inversion is obtained]

$$\omega_{kn} \to 0 \ \frac{\sin^2(\omega_{kn}t/2)}{\omega^2_{kn}} = \frac{t^2}{4}$$

and probability will be min.

15

(P_{nk}) when

$$\frac{\sin^2(\omega_{kn} t/2)}{\omega^2_{kn}} = 0$$

It will be zero.

These are the points of minima :-

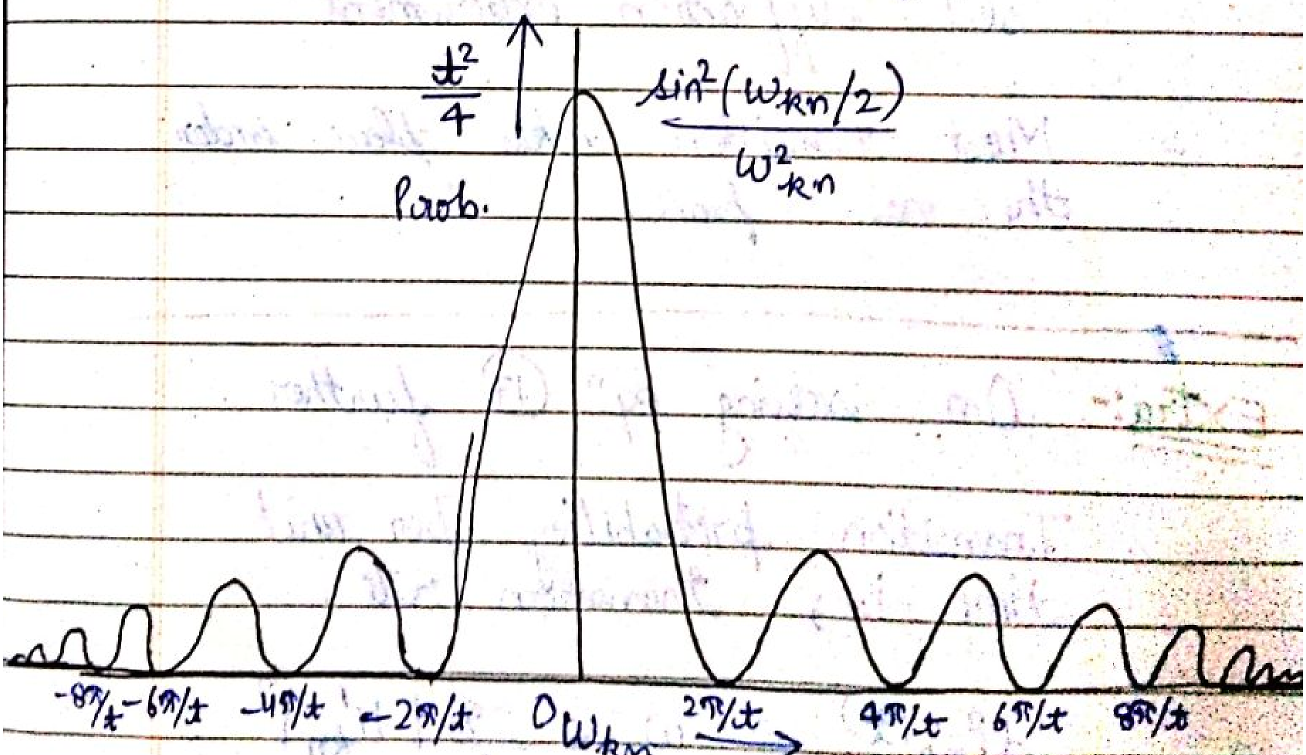
$$\Rightarrow \omega_{kn} = 0, \pm\frac{2\pi}{t}, \pm\frac{4\pi}{t}, \pm\frac{6\pi}{t} + \cdots$$

$$\cdots + \frac{2\pi n}{t}$$

$$\omega_{kn} = 0 \Rightarrow \omega_k - \omega_n = 0 \Rightarrow \omega_k = \omega_n$$

$\Downarrow$

all particles are in ground state.



(Single slit diffraction)

Prob. vs. frequency curve

From the above it is clear that :-

1. Height of main peak is propotional to $\frac{t^2}{4}$ i.e; $\propto t^2$

2. Width of peak is propotional to $\frac{1}{t}$ i.e. $\propto \frac{1}{t}$

3. Area under the curve is propotional to 't' i.e; time of application of perturbation.

4. Graph is similar to a single slit diffraction experiment.

5. Most transition takes place under the main peak.

Extra:- On solving eq^n 17 further

Transition probability per unit time i.e; transition rate.

$$\frac{dP_{nk}(t)}{dt} = \omega = \frac{2\pi}{\hbar} |H'_{kn}|^2 \delta(E_k - E_n)$$

⇒ Phonon momentum.

or $\frac{2\pi}{\hbar}\left|\int \psi_k^* H' \psi_n d^3\vec{r}\right|^2 \delta(E_k - E_n)$

unperturbated energy levels

$\delta(E_k - E_n)$ = Energy conservation

$|H'_{kn}|$ or $\underbrace{\left|\int \psi_k^* H' \psi_n d^3\vec{r}\right|}_{\text{Schroedinger.}}$ or H'_{kn}

matrix matrix

or $\underbrace{\langle k|H'|n\rangle}_{\text{Dirac and Heisenberg}}$ = momentum

i.e; Momentum and energy both remain conserved during the transition

For e.g. Let us consider two atoms system then according to the law of conservation of Energy

$$E(k) + E_1(k_1) = E'(k') + E_1'(k_1')$$

where $\vec{k}$ and $\vec{k_1}$ are wave vector

and $\vec{k} + \vec{k_1} = \vec{k'} + \vec{k_1'}$

$$\hbar\vec{k} + \hbar\vec{k_1} = \hbar\vec{k'} + \hbar\vec{k_1'}$$

$\hbar k$ = conjugate momentum
or
momentum or phonon momentum

This is momentum conservation.

Thus the transition rate gives energy and momentum conservation.
Hence, a constant time dependent perturbation, neither removes energy from the system nor supplies energy to it.
It simply causes energy and momentum conserving transitions.